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OPTIMIZATION OF CONVENTIONAL PASSIVE SONAR DETECTION SYSTEMS

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ABSTRACT

This report examines three methods for maximizing the output signal-to-noise ratio of a conventional passive detection system consisting of a fixed array, non-adaptive beam forming, and square-law detection. The performance of a non-optimal system is compared to each of the three optimum configurations, and the results suggest how an unsophisticated, sub-optimal system, which has only marginally poorer performance than the optimum systems, can be designed.

The specification of such a sub-optimal system depends greatly upon having intelligent estimates of the frequency domain behavior of the received signal; however, this information appears only implicitly in the solution, not explicitly as in the specification of an Eckart filter. Thus, one avoids the essentially insurmountable practical problems of attempting to build a system matched to the signal characteristics (the Eckart filter), while at the same time utilizing the expected variation in signal spectra to specify a realizable system whose performance is only slightly inferior to an Eckart system.

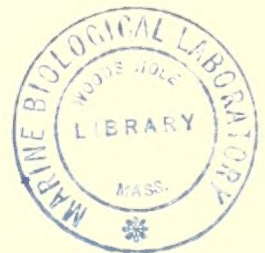Prepared by: _Robert J. Whelchel_
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TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

LIST OF SYMBOLS AND ABBREVIATIONS

It is recognized that there will be readers of this report who are not interested in the mathematical details, but only in the results and their significance. To assist these readers, the following list of important symbols and abbreviations is included; references are made to pertinent equations in the text. This list, combined with spot references to the body of the text, should allow the interested reader to obtain an understanding of the problem and results by reading sections 1, 2, and 8.

w.r.t. - "with respect to"

SNR - ubiquitous throughout the text, denotes "signal-to-noise ratio" and is always written on a power, not voltage, basis at the point of definition or measurement; consequently, dbs are obtained by taking 10 log of this quantity.

SNRo - used in two senses: (1) in a general sense, it denotes the SNR out of a square-law detector averager (Eqns. 9 and 10) and (2) in numerical results, it denotes the output SNR of a conventional band-limited pre-whitening system (Eqn. 16).

SNRo' (∞) - system output SNR for true Eckart filter (Eqns. 26 and 29): the prime denotes Eckart, ∞ denotes the infinite bandwidth associated with true spectral matching.

SNRo' - system output SNR for bandlimited Eckart filter. (Eqn. 34).

SNRo' (ξ_0) - system output SNR for mismatched Eckart filter. (Eqn. 43): ξ_0 is the matching parameter (see below).

$S_i(R,f)$, $S_i(f)$ - denotes the signal power spectrum at the input to the shaping filter. (Eqn. 1, Fig. 2) Because of its dependence upon propagation loss, the range variable, R, is initially included, but suppressed after section 2 since the results presented here do not depend upon it.

$N_i(f)$ - noise power spectrum at the input to the shaping filter (Eqn. 2, Fig. 2).

$S(R,f)$, $S(f)$ - signal power spectrum at the input of the square-law device (Eqn. 13, Fig. 2)

$\left| \Gamma(f) \right|^2$ - square of the transfer function of the shaping filter (Eqn. 13, Fig. 2).

$\wedge$ - this symbol over a power spectrum, e.g. $\hat{S}(f)$, denotes single sided spectra (Eqn. 10). Generally, double sided spectra are used in derivations while single sided spectra are used to obtain numerical results. See Appendix A.

α - Power of the frequency dependence of the signal spectra, i.e., $\hat{S}_i(f) = Af^{\alpha}$ (Eqn. 12).

$N(f)$ - noise power spectrum at the input of the square-law device (Eqn. 5, Fig. 2).

β - power of the frequency dependence of the noise spectra, i.e., $\hat{N}_i(f) = Bf^{\beta}$ (Eqn. 12).

$\xi \equiv \beta - \alpha$ - referred to as the relative signal spectral slope; the signal spectra has a slope of $-10 \log 2^{\xi}$ db/oct. w.r.t. the input noise spectra (Eqn. 15).

ξ_0 - refers to the mismatched Eckart filter: ξ_0 is the signal slope designed for, which is not necessarily the actual signal slope ξ (Eqn. 41 and 42).

T - the integration time associated with the output smoothing filter (Eqn. 9 and preceding discussion)

f_1 - lower frequency limit of operating band; spectra are all assumed to be zero below this value (Eqn. 12).

W - the bandwidth of bandlimited systems: thus, the operating band extends from f_1 to $f_1 + W$ (Eqn. 12).

$\mu \equiv \dfrac{W}{f_1}$ - normalized bandwidth variable (Eqn. 17).

μ_o - the value of bandwidth, μ, which maximizes the output SNR of a conventional pre-whitening bandlimited system for each signal slope ξ. (Eqn. 18 and Fig. 4).

$\gamma_\xi(\mu)$ - function proportional to the output SNR of a conventional bandlimited pre-whitening system of bandwidth μ and with input relative signal spectral slope of ξ. The quantity $\gamma_\xi(\mu)/\gamma_\xi(\mu_o)$ is the ratio of the output SNR of a pre-whitening system of bandwidth μ to the output SNR of a pre-whitening system with optimum bandwidth μ_o (Eqns. 16 and 17, Fig. 4).

δ_ξ - multiplicative factor dependent upon ξ alone, which is indicative of the performance of a bandlimited pre-whitening system w.r.t. a true Eckart filter (Eqns. 30 and 31).

$\varkappa_\xi(\mu)$ - a function which is the ratio of the output SNR of a bandlimited Eckart filter of bandwidth μ to the output SNR of a true Eckart filter with input signal spectral slope ξ (Eqns. 34, 35, and 36).

I. PRELIMINARY DISCUSSION

To some, the optimization of a conventional system may seem paradoxical since the word conventional tends to imply non-optimal processing; but the concept of optimization is multi-faceted, and the term remains ambiguous until system configuration constraints and criteria of optimality are specified. In this paper, an optimum system is one which maximizes the system output signal-to-noise ratio (SNR): from the theoretical viewpoint of statistical decision theory, this criterion is very limited, and a more satisfactory approach would be to utilize the Bayes decision rule (1, 2, 3, 4,); however, most engineers are more familiar with the concept of SNR than with Bayes decision rules, and since, for the analytical assumptions imposed upon the following work, the two criteria lead to the same optimized configurations,* the former method is chosen.

Further, optimization is not an end unto itself. Since optimum systems can never be fully realized in practice, one of the major purposes of mathematical optimization is to determine the limit on achievable performance under realistic constraints. In view of this, the primary emphasis of this paper is placed upon the evaluation of a sub-optimum system, i.e., the relative cost (in dB's of output SNR) for utilizing a non-optimum system in place of an optimum system.

The fundamental constraint imposed upon the following work is that the system configuration remains as shown in Figure 1 for all degrees of optimization. The system consists of an array of hydrophones, a simple delay and sum beamformer (with constant multiplicative spatial shading coefficients allowable), and a signal processing unit consisting of a square-law detector. Since such a configuration forms the basis for most contemporary systems, it will be denoted as a conventional system. In contrast to this conventional system are the various "adaptive" systems which continuously modify hydrophone data on the basis of past information (5, 6, 7, 8); such systems are still in the conceptual state, and are excluded from this analysis.

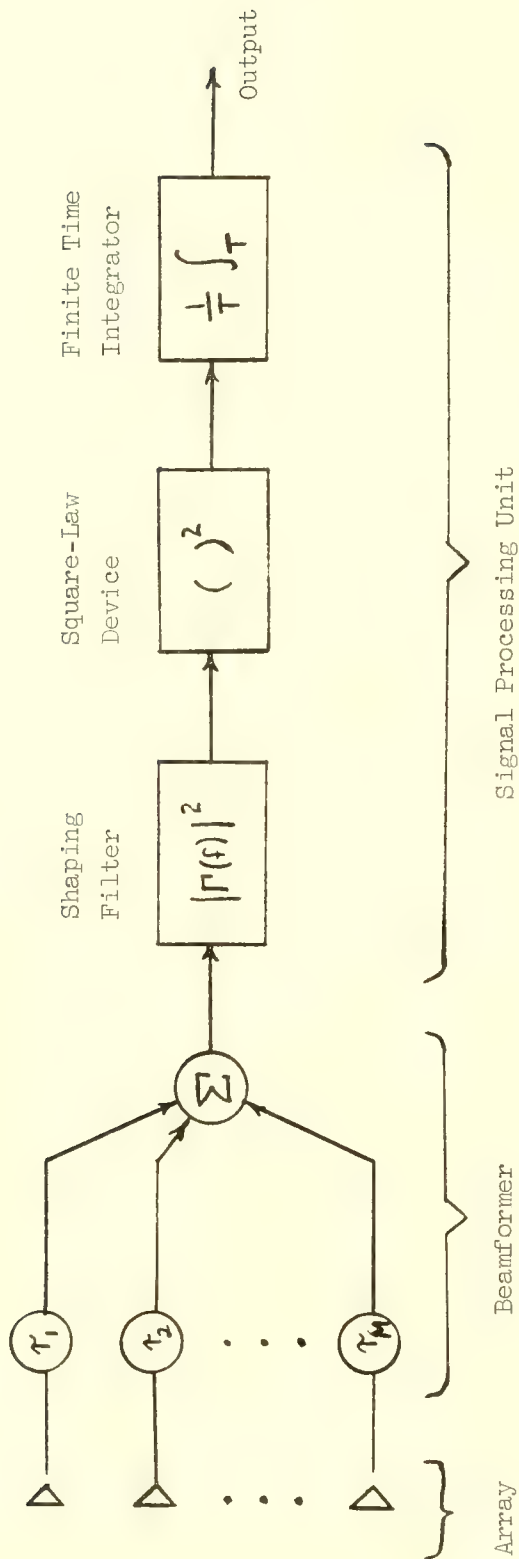*See Appendix D (1) for amplification of this point.

Figure 1 Conventional Passive Detection System

The optimization of the conventional system in Figure 1 is further restricted to the optimal specification of the shaping filter, $|\Gamma(f)|^2$; the array configuration and beamformer design are assumed fixed, and square-law detection is used throughout. In the non-optimal system which is evaluated, the shaping filter is used only to whiten the noise and bandlimit the input to the square-law detector; the performance of this system is compared to three different optimum systems, the three optimizations resulting from different sets of imposed contraints.

In the first optimization, it is assumed that the designer has no a priori knowledge of the signal spectral characteristics, except, perhaps, a vague knowledge of the expected extreme limits. In this case, it is generally assumed that the best one can do is to whiten the noise and appropriately bandlimit the input;** thus, this optimal system differs from the non-optimal system only to the extent that the bandwidth is (somewhat artifically) optimized. In fact, the procedure followed is to evaluate the performance of the non-optimal system as a function of band-width, and to denote that the bandwidth which maximizes the output SNR for a particular (but unknown to the designer) signal spectrum as optimum. The procedure may appear strained and artificial until it is remembered that the main purpose is not to determine a strictly mathematical opti-mum system but to evaluate the relative performance of the non-optimal system. The results will show the cost, in dbs of output SNR, for utilizing an incorrect bandwidth for a particular signal spectrum; per-haps more important physically, the results also show how the performance of a fixed bandwidth non-optimal system deviates from optimum as the input spectrum varies between certain limits.

The second optimization makes use of the well known Eckart filter (6, 8, 9). In this second case, it is assumed that the designer does have prior knowledge of the signal spectrum; this knowledge can be effectively utilized to determine a form of the shaping filter which maximizes the output SNR. Such a filter specification depends upon both signal and noise spectral characteristics, and can be viewed as the

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**For discussion, see Appendix D (2)

passive equivalent of the matched filter for active systems (1, 10, 11).
The non-optimum system performance, again as a function of bandwidth, is
compared to an optimal system using such an Eckart filter.

The third optimization is a modification of the Eckart result.
The assumed signal and noise spectra all approach zero asymptotically as
the frequency approaches infinity; thus, mathematically, the Eckart
filter has infinite bandwidth. Physical considerations, as well as
curiosity, leads one to examine the effect of terminating the high
frequency response while retaining the Eckart filter characteristic
over the finite pass-band. The comparison made in this third case is
thus between a bandlimited Eckart filter and the bandlimited non-optimal
conventional system, where both systems are restricted to having the
same bandwidth.

The three cases illustrate the influence of various filter
parameters upon performance. The first case demonstrates the effect of
bandwidth; the second, what is to be gained from having and utilizing
signal information to its fullest extent; the third case combines the
first two, and demonstrates the influence of signal knowledge in a system
which similarly must be bandlimited.

Thus, the results provide a quantitative estimate of the rela-
tive value of signal spectral knowledge; i.e., how much performance can
be improved by the insertion of an Eckart filter in place of a simple
bandlimiting pre-whitener.

In order for one to have a keener appreciation for these results, the
next section briefly outlines the tactical passive sonar situation and
those factors which influence the performance of a conventional detec-
tion system.

II. THE PASSIVE SONAR SITUATION

For the purposes of this analysis, the passive sonar situation can be depicted as shown in Figure 2. The quantities on the left are generally beyond the control of the sonar engineer, and may be considered as inputs. The target which is sought radiates noise of spectral content $T(f)$. The signal spectrum received at the detecting platform is $T(f)$ as modified by the propagation loss $H(R,f)$. Masking the target radiated noise are both environmental and platform generated (self) noise; it is assumed that these two noise processes are independent so that their power spectra combine additively. This total noise is reduced by the array gain, so that the resultant signal and noise spectra into the shaping filter are*

$$S_i(R,f) = T(f) H(R,f) \tag{1}$$

$$N_i(f) = \frac{N_S(f) + N_E(f)}{A(f)} \tag{2}$$

The spectral characteristics of the quantities on the right of equations (1) and (2) can, of course, be rather complex; however, in regions of interest, the following trends are generally exhibited:**

$T(f)$: spectral slope of approximately -5db/oct.

$H(\dot{R},f)$: highly variable spectral slope, depending upon mode of propagation, range, and environmental conditions; typical slopes vary between 0db/oct and -20db/oct.

$N_S(f)$, $N_E(f)$: spectral slope of approximately -5db/oct.

$A(f)$: spectral slope of approximately +6db/oct for planar and volumetric arrays, +3db/oct for line arrays***

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*This formulation assumes the target is on the major response axis of the beam; an off axis target would naturally have its spectrum modified by the frequency dependence of the beam response.

**These values are presented only to give a physical feeling for the problem being treated. They can be ignored from a mathematical standpoint, since the problem can be viewed simply as an optimization exercise. That the stated trends are representative can be determined by examining references 12 and 13.

***Arrays assumed to be in their aperture response region, i.e., less than half wave length spacing of the elements.
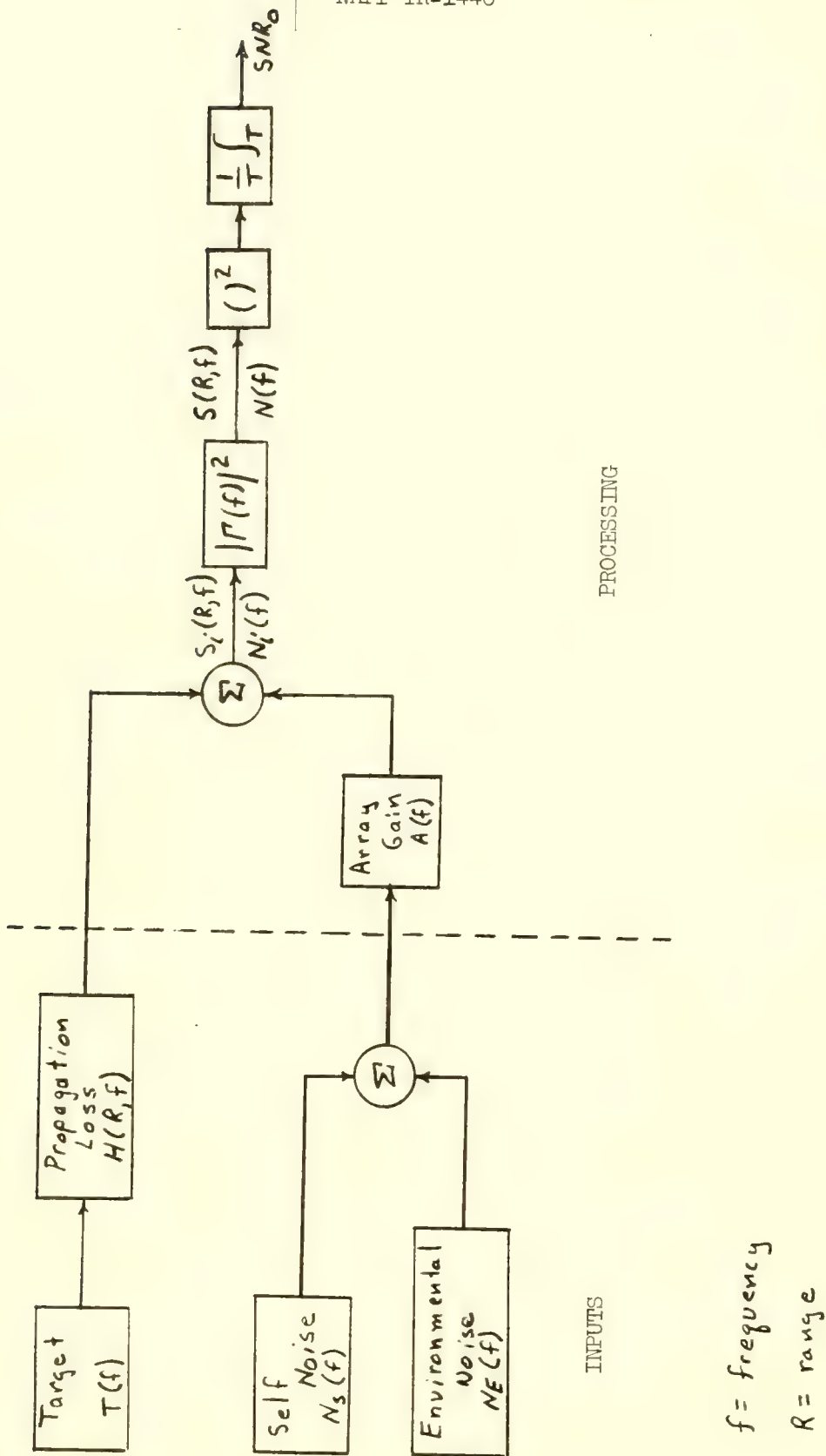
Figure 2   Passive Sonar Situation

$f = $ frequency

$R = $ range

6

   As implied from the preceding, the spectral characteristics
of $S_1(R,f)$ exhibit a higher degree of variability than those of $N_1(f)$
by virtue of the propagation loss, $H(R,f)$. Consequently, it is only
natural to examine what effect this variation has upon the performance
of a passive detection system.

III. DERIVATION OF CONVENTIONAL SYSTEM EQUATION

In order to proceed with the performance evaluation, it is necessary to determine the relationship between the system output SNR and the signal and noise spectra into the square-law device. Compared to the remainder of this paper, this section is rather mathematical. The reader should have a basic understanding of Fourier transforms and their manipulation as well as a familiarity with the fundamental concepts of signal processing; the second chapter of Woodward's monograph provides an adequate coverage of this material (10). Those not so endowed can skip to the next section with little loss in continuity.

This section is concerned with only two blocks of the processing functions shown in Figure 2; these are the square-law device and the output smoothing filter, shown in Figure 3. The input, $x(t)$, consists of either signal plus noise or noise alone:

$$x(t) = \begin{cases} s(t) + n(t) & (3a) \\ \quad\text{or} \\ n(t) & (3b) \end{cases}$$

Assume that $s(t)$ and $n(t)$ are members of a zero mean stationary Gaussian random process. Then the correlation function at the output of the square-law device is (14, p. 255).

$$R_y(\tau) = \sigma_x^4 + 2 R_x^2(\tau)$$

Further assume that $s(t)$ is statistically independent of $n(t)$, and that they have variances σ_s^2 and σ_N^2; then

$$R_y(\tau) = \begin{cases} (\sigma_s^2 + \sigma_N^2)^2 + 2[R_s^2(\tau) + R_n^2(\tau) + 2 R_s(\tau) R_n(\tau)] & (4a) \\ \quad\text{or} \\ \sigma_N^4 + 2 R_n^2(\tau) & (4b) \end{cases}$$

Realizing that the power spectral density of a random process is the Fourier transform of its correlation function, and that multiplication in the τ domain corresponds to convolution in the frequency domain, the power spectral density of $y(t)$ is

$$S_y(f) = \begin{cases} (\sigma_s' + \sigma_N^2)^2 \, \delta(f) + 2[S(f) \otimes S(f) + N(f) \otimes N(f) \\ \qquad\qquad\qquad\qquad\qquad + 2S(f) \otimes N(f)] & (5a) \\ \quad\text{or} \\ \sigma_N^4 \, \delta(f) + 2 N(f) \otimes N(f) & (5b) \end{cases}$$
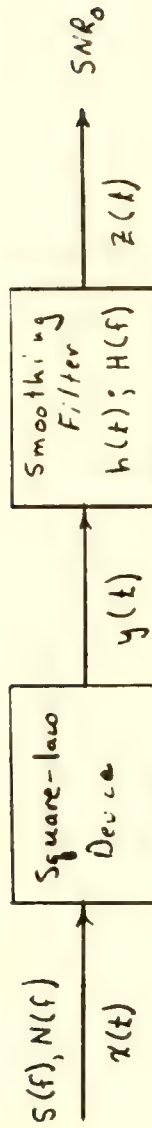
8

Figure 3 Power Detector

where $\delta(f)$ is the Dirac delta function, $S(f)$ and $N(f)$ are the signal
signal and noise power spectra into the square-law device,* and the symbol
⊠ denotes convolution.

The wave form $y(t)$ is averaged by a filter with transfer function
$H(f)$; thus the output power spectrum is

$$S_z(f) = Sy(f) \left| H(f) \right|^2 \tag{6}$$

The results of equations (5) and (6) contain the information
needed to determine the system output SNR, which must now be defined. The
output signal of interest is the shift in the average (d-c) voltage
level of the output due to the presence of signal. Assuming that the
input signal and noise spectra do not contain impulses (i.e., ignoring
the effect of lines), and assuming $\left| H(0) \right| = 1$, the output d-c voltage with
signal present is $\sigma_s + \sigma_N^2$ (the $\delta(f)$ term from equation 5a). Since the d-c
level with noise alone is σ_N^2 (equation 5b), the output signal voltage of
interest is σ_s^2. Returning to a power basis in order to write SNR's,
the output signal power is σ_s^4.

The continuous portion of $S_z(f)$ is the "ripple", or noise,
present at the smoothing filter output which tends to obscure the shift in
d-c level due to the presence of signal. There are two possible definitions
of output noise: one would involve using the continuum portion of the
spectrum in equation 5a, i.e., the ripple due to the presence of both
signal and noise at the input; the other involves the continuum portion of
the spectrum in equation 5b, i.e., the ripple due to the presence of noise
alone at the input. For a complete description of processor performance,
both quantities are needed; however, in most passive sonar situations of
interest, the input SNR<<1, and, in this case, the presence of a signal
has a negligible effect upon the output variation. Because of this effect,
as well as the resulting mathematical simplicity and the degree of arbi-
trariness associated with any definition of SNR, the output noise is

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*The range variable, R, has been suppressed since it exerts no influence
upon the following results.

defined to be the output variation due to noise alone at the input.*
Thus, the output SNR is, from equations 5 and 6,

$$SNR_o = \sigma_s^4 \left\{ 2 \int_{-\infty}^{\infty} [N(f) \otimes N(f)] \, |H(f)|^2 df \right\}^{-1} \tag{7}$$

This expression can be simplified a great deal by one further
assumption often satisfied in practice. Assume that the output filter
bandwidth is very narrow compared to the input bandwidth of the system;
under these conditions, the auto-convolution of $N(f)$ will have a very broad
peak in the region of $f=0$ when compared to $|H(f)|^2$. The value of this peak
will be the auto-convolution evaluated at $f=0$ i.e.,

$$\int_{-\infty}^{\infty} N^2(\xi) \, d\xi$$

since $N(-\xi) = N(\xi)$, i.e., the power spectra of real functions are real and
even. Utilizing this assumption that the output filter response is suffi-
ciently narrow so that $|H(f)|^2 \simeq 0$ before the value of $N(f) \otimes N(f)$ has varied
appreciably from its value at $f=0$, and further recognizing σ_s^2 as the input
signal power, equation 7 can be approximated by

$$SNR_o = \frac{1}{2} \frac{\left[\int_{-\infty}^{\infty} S(f) \, df \right]^2}{\left[\int_{-\infty}^{\infty} N^2(f) \, df \right] \left[\int_{-\infty}^{\infty} |H(f)|^2 df \right]} \tag{8}$$

Equation 8 is a rather general expression for the output SNR of a square-law
detector-averager. The only restrictive assumptions used to derive this
equation are

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*This definition of output SNR corresponds to the following statistical
definition:

$$SNR_o = \frac{\left[\overline{Z(S+N)} - \overline{Z(N)} \right]^2}{\sigma_Z^2(N)}$$

where $\overline{Z}$ is the output mean, σ_Z^2 is the output variance, and N and S + N
denote the presence of noise alone or signal plus noise, respectively.

(1) the input signal and noise are members of stationary Gaussian random processes, independent of one another, and

(2) the input noise is "wide band" compared to the output filter.

For the remainder of this paper, the output smoothing filter is chosen to be a perfect, finite time averager with impulse response

$$h(t) = \begin{cases} 1/T & 0 < t < T \\ 0 & elsewhere \end{cases}$$

Then (10, 15)

$$\int_{-\infty}^{\infty} |H(f)|^2 df = \int_{-\infty}^{\infty} [sinc(Tf)]^2 df \qquad \frac{1}{T}$$

Thus, for this output filter, the expression for the output SNR may be written as

$$SNR_o = \frac{T}{2} \frac{\left[\int_{-\infty}^{\infty} S(f) df \right]^2}{\int_{-\infty}^{\infty} N^2(f) df} \qquad (9)$$

If single sided power spectra, $\hat{S}(f)$ and $\hat{N}(f)$, are used in place of the double sided $S(f)$ and $N(f)$, the resulting expression is

$$SNR_o = T \frac{\left[\int_{0}^{\infty} \hat{S}(f) df \right]^2}{\int_{0}^{\infty} [\hat{N}(f)]^2 df} \qquad (10)$$

as shown in Appendix A.

If the noise is white over a band W, it is a straightforward process to show that equation 10 (or 9) reduces to

$$SNR_o = TW \left(\frac{S}{N} \right)^2 \qquad (11)$$

where S and N are the signal and noise power (total power, <u>not</u> spectral levels) into the square-law device.*

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*See Appendix B for a generalization of equation (11).

IV. CASE 1: BANDWIDTH OPTIMIZATION IN WHITE NOISE

In order to obtain numerical results, some functional form must be assumed for the input signal and noise spectra, $S_1(f)$ and $N_1(f)$ in Figure 2. Since many of the quantities in the sonar equation can be conveniently represented by a db/octave slope over the region of interest, the following dependence is assumed:

$$\hat{S}_i(f) = \begin{cases} A f^{\alpha} & f_1 < f < f_1 + W \\ 0 & \text{else where} \end{cases} \qquad (12a)$$

$$\hat{N}_i(f) = \begin{cases} B f^{\beta} & f_1 < f < f_1 + W \\ 0 & \text{else where} \end{cases} \qquad (12b)$$

The spectra are thus assumed to be sharply bandlimited, and to be characterized by constant db/octave slopes within the band $(f_1, f_1 + W)$. This is an oversimplified representation, particularly for $S_1(f)$ because of the propagation effects mentioned earlier; however, when tempered by reason, the results from such spectra shed light upon passive system performance.

The spectra into the square-law device are

$$\hat{S}(f) = \hat{S}_i(f) |\Gamma(f)|^2 \qquad (13a)$$

$$\hat{N}(f) = \hat{N}_i(f) |\Gamma(f)|^2 \qquad (13b)$$

To whiten the noise, the shaping filter must be of the form*

$$|\Gamma(f)|^2 = f^{-\beta} \qquad f_1 < f < f_1 + W \qquad (14)$$

so that

$$\hat{S}(f) = A f^{\alpha - \beta} = A f^{-\xi} \Big\} \quad f_1 < f < f_1 + W \qquad (15a)$$

$$\hat{N}(f) = B \qquad (15b)$$

where $\xi = \beta - \alpha$ is the input signal spectral slope with respect to the input noise spectral slope, i.e., the slope of the signal spectrum into the square-law device.

The direct application of equation (10) to the spectra in equation (15) results in

$$SNR_o = \begin{cases} T \left(\frac{A}{B}\right)^2 f_1^{1-2\xi} (1-\xi)^{-2} \gamma_\xi(\mu) & , \xi \neq 1 \qquad (16a) \\ T \left(\frac{A}{B}\right)^2 f_1^{-1} \gamma_1(\mu) & , \xi = 1 \qquad (16b) \end{cases}$$

_____
*The arbitrary multiplicative constant is chosen to be unity for simplicity.

where

$$\gamma_\xi(\mu) = \frac{1}{\mu}\left[(1+\mu)^{1-\xi}-1\right]^2 \quad, \quad \xi \neq 1 \qquad (17a)$$

$$\gamma_1(\mu) = \frac{1}{\mu}\left[\ln(1+\mu)\right]^2 \quad, \quad \xi = 1 \qquad (17b)$$

and $\mu = W/f_1$ is the normalized bandwidth variable, which gives the system bandwidth as a multiple of f_1. Thus, $\mu = 1$ corresponds to an octave band.

In any given sonar situation, with an assumed constant relative signal spectral slope ξ, T, A, B, f_1, and ξ are all constant. Thus, the $\gamma_\xi(\mu)$ describe the relative SNR performance as a function of bandwidth. From the expressions in equation 17, it can be seen that the possibility exists for $\gamma_\xi(\mu)$ to have a relative maximum at some value $\mu = \mu_0$, depending upon the value of ξ. Considering the $\xi \neq 1$ case, setting $\frac{d\gamma_\xi}{d\mu} = 0$ results in the following equation for determinant μ_0:

$$1 + \mu(2\xi - 1) = (1+\mu)^\xi \quad, \quad \xi \neq 1 \qquad (18)$$

Thus if $\xi = 2$, $\mu_0 = 1$; consequently, for a relative signal spectral slope of -6db/oct, the bandwidth which maximizes the output SNR is an octave wide.

there does not always exist an optimum finite bandwidth, if $\xi = 0$, i.e., the signal spectrum into the square-law device is also white, then equation 18 results in $\mu_0 = 0$, which is obviously not the relative maximum sought. Referring to equation (11), if both signal and noise are white, then the SNR into the square-law device ($\frac{S}{N}$ in Eqn. 11) is a constant as W is varied, and equation (11) indicates that the output SNR will increase linearly with W. Thus, the bandwidth which would maximize the output SNR in this case, $\xi = 0$, would be infinitely wide. Naturally, such a spectrum will not occur physically as various limiting mechanisms will eventually cause the high frequency response to fall off; however, mathematically, no finite μ_0 exists.

Looking at equations (11) and (18), it can be seen that an infinite bandwidth solution will result until some critical value of ξ, $\xi_0 > 0$, is used for the relative signal spectral slope. Thus, in range $0 < \xi < \xi_0$, even though the signal spectrum is falling off compared to the noise spectrum, and, as a consequence, the SNR into the square-law device is decreasing as W is increased, signal processing

(the TW portion of Eqn. 11) more than compensates for the decreasing input SNR. Intuitively, one would suspect the value of ξ_0 to be $\frac{1}{2}$ (corresponding to a relative signal spectral slope of -1.5db/oct); for once, intuition proves correct. For $\xi = \frac{1}{2}$, equation 18 shows that no relative maximum exists, and the infinite bandwidth solution again results. Appendix C demonstrates that $\xi = \frac{1}{2}$ is the critical value, and that for values of ξ incrementally larger than $\frac{1}{2}$, finite bandwidth solutions result.

One could use equation 18 to determine optimum bandwidths for various values of $\xi(\xi \neq 1, \xi > \frac{1}{2})$; however, it is not only the optimum value of bandwidth that is of concern, but also the penalty that is paid for not operating at the optimum. This is simply the evaluation of the non-optimum system discussed in Section 1. Such an evaluation can be accomplished by examining a normalized $\gamma_\xi(\mu)$ as a function of μ for various values of ξ in the region where optimum bandwidths exist. This normalized $\gamma_\xi(\mu)$, given by $\gamma_\xi(\mu)/\gamma_\xi(\mu_0)$, is plotted in Figure 4 for ξ equal to 1 through 6 in integer values.

Referring to equation 16, it can be seen that $\gamma_\xi(\mu)/\gamma_\xi(\mu_0)$ is simply the ratio of the output SNR at a normalized bandwidth μ to the output SNR for the optimum bandwidth, μ_0, corresponding to the particular value of ξ. The quantity $\gamma_\xi(\mu)/\gamma_\xi(\mu_0)$ can thus be conveniently referred to as the relative output SNR since it measures the SNR out of a non-optimum system relative to the output SNR that would be obtained had an optimum bandwidth been employed.

The graphs in Figure 4 are intuitively satisfying since the peaks are rather broad, indicating little loss associated with picking a non-optimum band. Often, an octave band ($\mu = 1$) is used in acoustic analysis systems. Figure 4 shows that for signal spectra into the square-law device with slopes between -3db/oct and -18db/oct, the loss in output SNR associated with an octave band is less than 3db. Thus, Figure 4 would indicate that an octave is a good compromise band. It must be stressed, however, that this section has dealt with extremely simple, and consequently, somewhat artificial, spectral characteristics. The results are enlightening and meaningful, but simply in pointing the way toward a solution; they do not provide a comprehensive answer to the
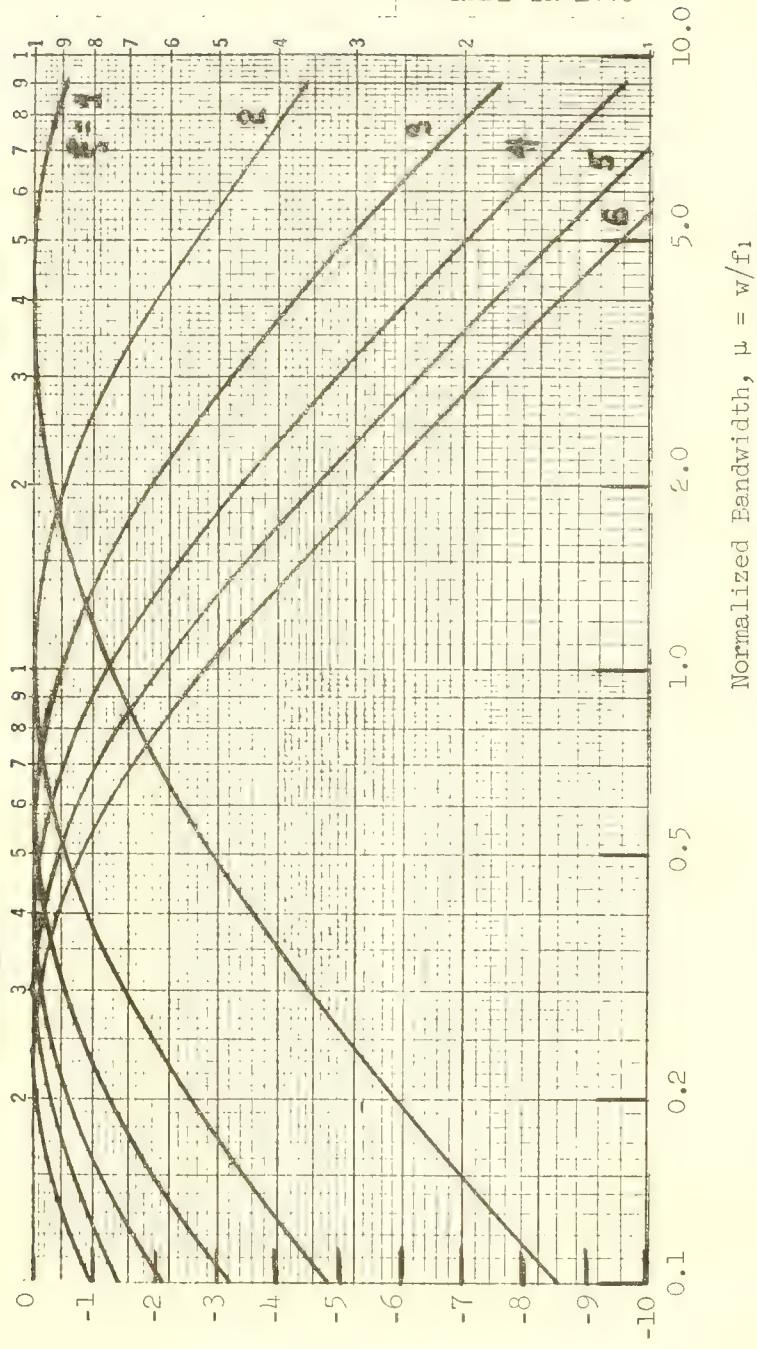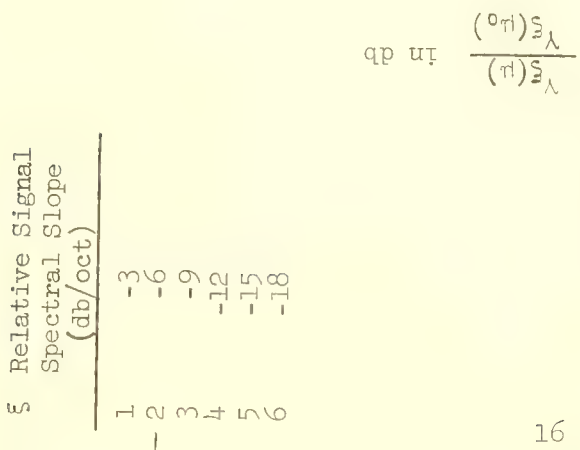
Figure 4   Relative Output SNR for Bandlimited Prewhitening System

Normalized Bandwidth, $\mu = w/f_1$

$\dfrac{\gamma_\xi(\mu)}{\gamma_\xi(\mu_o)}$ in db

16

ξ	Relative Signal Spectral Slope (db/oct)
1	-3
2	-6
3	-9
4	-12
5	-15
6	-18

	Approximate Optimum Values of μ
ξ	μ_o
1	4.2
2	1.0
3	0.57
4	0.40
5	0.30
6	0.24

passive sonar detection problem. Besides the simplified spectral characteristics, no mention has been made about determining f_1, basically, specifying the location of the operating band. Such a specification is strongly dependent upon the basic sonar inputs of Figure 2 -- the target, noise, and propagation characteristics -- as well as physical limitations upon the array size and available equipment space for beamforming and processing. The consideration of all these factors would lead to a total system design, which is well beyond the scope and intent of this article; however, some further general comments about the influence of these quantities upon system bandwidth specification will be made in the concluding section.

V. CASE 2: THE ECKART FILTER

The previous section examined what might be termed a theoretically crude optimization scheme: the shaping filter was constrained to be a noise pre-whitening filter only, and the optimization process was performed only on bandwidth. In this section, the optimization is more sophisticated, making use of signal spectral knowledge to determine the best form of shaping filter. The result is the well Eckart filter, given by

$$|\Gamma(f)|^2 = \frac{S_i(f)}{[N_i(f)]^2} \qquad (19)$$

To show that this is the optimum filter, equation 9 will be maximized by a common procedure (8): assume a general shaping filter response given by

$$\Gamma(f) = \Gamma_0(f) + \epsilon\gamma(f) \qquad (20)$$

where $\Gamma_0(f)$ is the optimum value of $\Gamma(f)$ sought, $\gamma(f)$ is an arbitrary frequency function, and ϵ is any scaler. Then the system output SNR is, from equations 9, 13, and 20,

$$SNR_0 = \frac{I}{2} \frac{\left[\int_{-\infty}^{\infty} S_i(f)(\Gamma_0 + \epsilon\gamma)(\Gamma_0^* + \epsilon\gamma^*)\,df\right]^2}{\int_{-\infty}^{\infty} N_i^2(f)[(\Gamma_0 + \epsilon\gamma)(\Gamma_0^* + \epsilon\gamma^*)]^2\,df} = \frac{N(\epsilon)}{D(\epsilon)} = \sigma(\epsilon) \qquad (21)$$

If Γ_0 is truly the optimum filter response, then $\frac{d\sigma}{d\epsilon}$ should be zero when ϵ is set equal to zero, i.e.,

$$\frac{d\sigma}{d\epsilon}\bigg|_{\epsilon=0} = 0 = \frac{I}{2D_0^2}\left\{ D_0\, 2\sqrt{N_0} \int_{-\infty}^{\infty} S_i(f)(\gamma\Gamma_0^* + \gamma^*\Gamma_0)\,df \qquad (22) \right.$$
$$\left. - N_0 \int_{-\infty}^{\infty} N_i^2(f)\, 2\,\Gamma_0\,\Gamma_0^*(\Gamma_0\gamma^* + \Gamma_0^*\gamma)\,df \right\}$$

where D_0 and N_0 represent the denominator and numerator of equation 21 evaluated at $\epsilon = 0$. Further manipulation results in

$$\int_{-\infty}^{\infty} (\gamma\Gamma_0^* + \gamma^*\Gamma_0)\left[S_i(f) - K N_i^2(f)|\Gamma_0|^2 \right] df = 0 \qquad (23)$$

where $K = \dfrac{\sqrt{N_o}}{D_o}$. The only way that equation 23 can be valid for all values
of $\gamma(f)$, which is an arbitrary function, is if

$$|\Gamma_o(f)|^2 = \frac{1}{K} \frac{S_i(f)}{N_i^2(f)} \tag{24}$$

Since K is a scalar constant, it acts merely as a scale factor which can
have no effect upon the optimum specification of $\Gamma(f)$; thus, it can be
arbitrarily chosen as unity, resulting in the specification of equation 19.

Using the Eckart filter for $|\Gamma(f)|^2$, the spectra into the square-
law device become

$$S(f) = \left[\frac{S_i(f)}{N_i(f)}\right]^2 \tag{25a}$$

$$N(f) = \frac{S_i(f)}{N_i(f)} \tag{25b}$$

By substitution into equation 10 (or 9), the output SNR takes the simple
form

$$SNR_o'(\infty) = T \int_{f_i}^{\infty} \left[\frac{\hat{S}_i(f)}{\hat{N}_i(f)}\right]^2 df \tag{26}$$

Utilizing the same spectral forms assumed in Section 4, but ex-
tending the high frequency response to infinity, the input signal and
noise spectra are

$$\hat{S}_i(f) = \begin{cases} Af^\alpha & , f > f_i \\ 0 & , f < f_i \end{cases} \quad \alpha < 0 \tag{27a}$$

$$\hat{N}_i(f) = \begin{cases} Bf^\beta & , f > f_i \\ 0 & , f < f_i \end{cases} \quad \beta < 0 \tag{27b}$$

we have

$$\frac{\hat{S}_i(f)}{\hat{N}_i(f)} = \frac{A}{B} f^{-\xi} \quad , f > f_i \tag{28}$$

where $\xi = \beta - \alpha$, as in Section 4.

Substituting equation 28 into 26 and integrating, the output SNR
for an infinite bandwidth Eckart filter is

$$SNR_o'(\infty) = T \left(\frac{A}{B}\right)^2 \frac{1}{(2\xi-1)} f_i^{1-2\xi} \quad , \xi > \frac{1}{2} \tag{29}$$

Referring to equation 16, the performance of the non-optimum conventional system with normalized bandwidth μ can be compared to the Eckart filter system. The relative loss in output SNR of the non-optimum system is conveniently expressed as

$$\frac{SNR_o}{SNR_o'(\infty)} = \delta_\xi \gamma_\xi(\mu) \qquad , \xi > \frac{1}{2} \qquad (30)$$

where δ_ξ is a multiplicative factor depending on ξ alone. amd given by

$$\delta_\xi = \begin{cases} \dfrac{(2\xi-1)}{(1-\xi)^2} & , \xi \neq 1 \\ \\ 1 & , \xi = 1 \end{cases} \qquad (31)$$

Thus, graphs of $SNR_o/SNR_o'(\infty)$ exhibit the same bandwidth dependence as Figure 4; the curve corresponding to each value of ξ is simply moved down by a factor of $\delta_\xi \gamma_\xi(\mu_0)$, which is a constant for each value of ξ. The results for equation 30 are shown in Figure 5. It can be seen that the additional "signal shaping" loss, i.e., the loss in addition to the bandwidth loss shown in Figure 4, amounts to some 1 to 2 db, with the higher loss being associated with the shallower relative signal spectral slope, $\xi = 1$. Figure 5 shows that an octave wide pre-whitening system loses between 1 and 4 db in output SNR compared to an Eckart "matched" system for relative signal slopes between -3db/oct and -18db/oct.
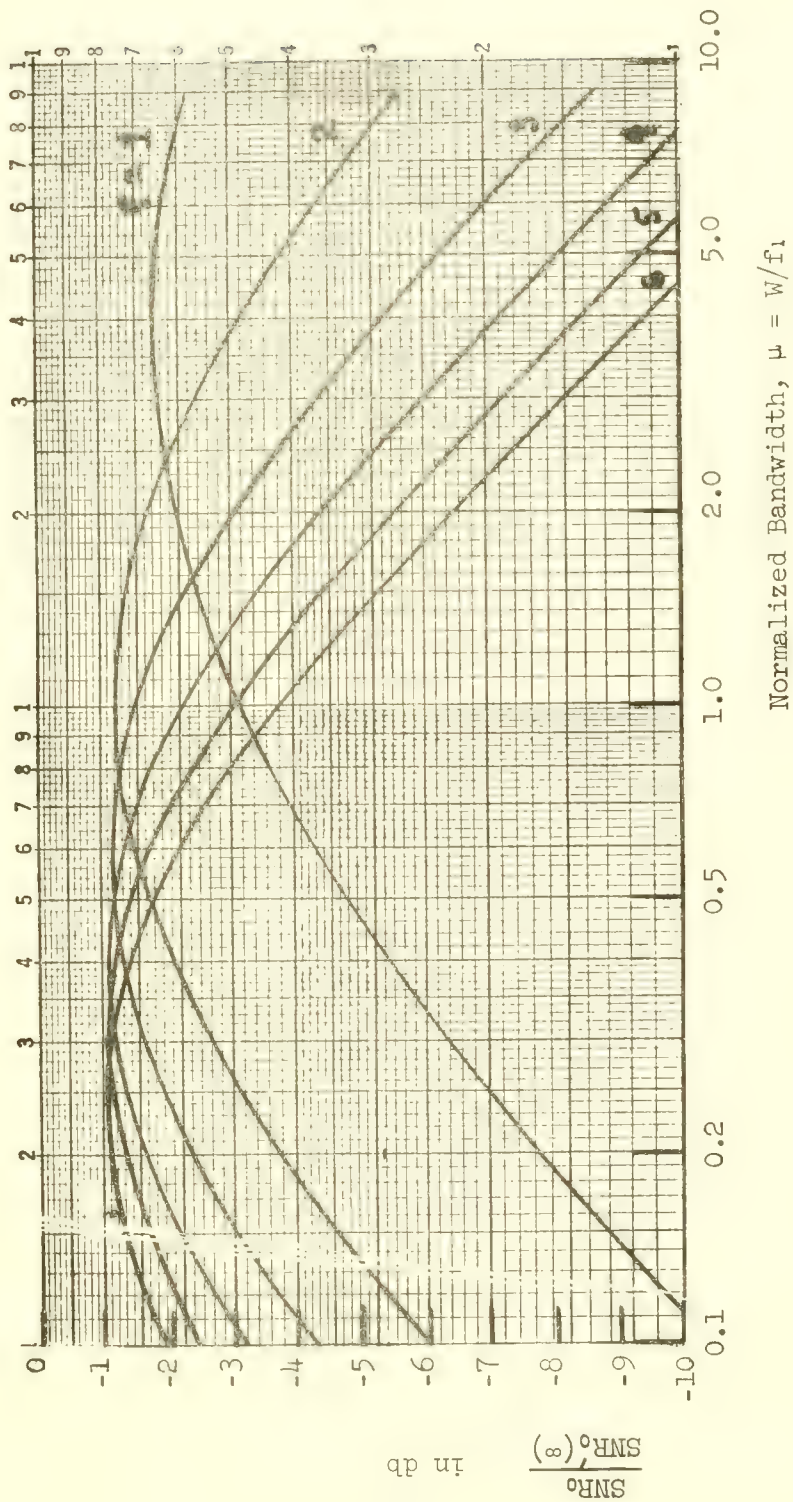
Normalized Bandwidth, $\mu = W/f_1$

Figure 5   Comparison of Bandlimited Prewhitening System (SNRo) to Eckart Filter System ($\text{SNR}_o'(\infty)$)

$\dfrac{\text{SNR}_o}{\text{SNR}_o'(\infty)}$ in db

21

VI.   CASE 3:   THE BANDLIMITED ECKART FILTER

The last optimization to be examined arises basically from curiosity: one wonders just how significant the higher frequencies are in the specification of the Eckart filter. In this section, the performance of both the infinite bandwidth Eckart filter and the pre-whitening, band-limited conventional system will be compared to a bandlimited Eckart filter, defined as

$$|\Gamma(f)|^2 = \begin{cases} \dfrac{\hat{S}_{i'}(f)}{[\hat{N}_i(f)]^2} & , \quad f_1 < f < f_1 + W \\ \\ 0 & , \quad elsewhere \end{cases} \tag{32}$$

The system output SNR for such a filter follows from equation 26:

$$SNR_0' = T \int_{f_1}^{f_1 + W} \left[\frac{\hat{S}_i(f)}{\hat{N}_i(f)} \right]^2 df \tag{33}$$

Utilizing the spectral characteristics from equation 27,

$$SNR_0' = T \left(\frac{A}{B} \right)^2 \frac{f_1^{1-2\xi}}{2\xi - 1} \eta_\xi(\mu) \tag{34}$$

where

$$\eta_\xi(\mu) = 1 - (1+\mu)^{1-2\xi} \qquad \xi \neq \tfrac{1}{2} \tag{35}$$

and the logarithmic solution for $\xi = \frac{1}{2}$ is ignored.

By comparison with equation 29, it can be seen that the ratio of the bandlimited Eckart filter to infinite bandwidth Eckart filter output SNR's is simply to factor in equation 35, i.e.,

$$\frac{SNR_0'}{SNR_0'(\infty)} = \eta_\xi(\mu) \tag{36}$$

The comparison to the pre-whitening bandlimited conventional system can be rapidly obtained from the following manipulation:

$$\frac{SNR_0}{SNR_0'} = \left[\frac{SNR_0}{SNR_0'(\infty)} \right] \left[\frac{SNR_0'(\infty)}{SNR_0'} \right] \tag{37}$$

Both factors on the right of equation 37 have already been determined, viz., from equations 30 and 36, so that

$$\frac{SNR_o}{SNR_c'} = \frac{\delta_\xi \, \gamma_\xi(\mu)}{\eta_\xi(\mu)} \tag{38}$$

Since we are only concerned with those cases for which optimum bandwidths exist for $\gamma_\xi(\mu)$, the value of ξ can be restricted to being greater than $\frac{1}{2}$, and no perplexing mathematical difficulties are encountered in the above manipulations.

Equation 36 is plotted in Figure 6; naturally, $\eta_\xi(\mu)$, the loss associated with bandlimiting the Eckart filter, approaches 0db as μ gets large, the approach being faster as ξ increases. The use of a bandlimited Eckart filter with $\mu = \mu_0$, the "optimum" bandwidth for each ξ from Figure 4, results in $\frac{1}{2}$ to 1 db of loss compared to the infinite bandwidth Eckart; recall that for the conventional pre-whitening system with $\mu = \mu_0$, the loss was from 1 to 2 db (Figure 5).

Equation 38 is plotted in Figure 7. Again the results agree with intuition. Where both the Eckart and pre-whitening filters have narrow bandwidths, they result in approximately the same performance. For large bandwidths, the factor $\eta_\xi(\mu)$ in equation 38 approaches unity, and the bandlimited Eckart out-performs the pre-whitening system by the same amount as the infinite bandwidth Eckart shown in Figure 5.
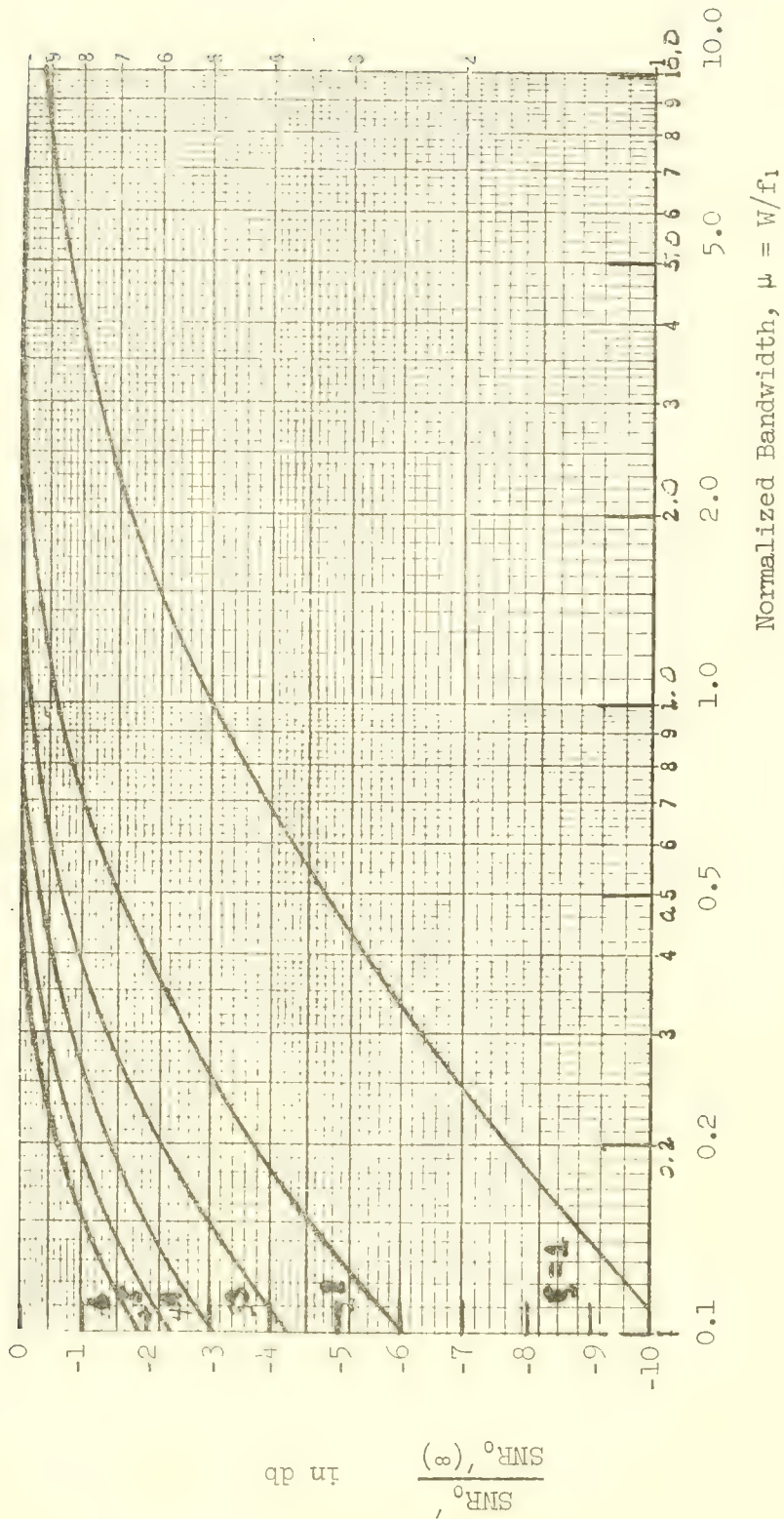
Figure 6 Comparison of Bandlimited Eckart System (SNR$_0$') to Infinite
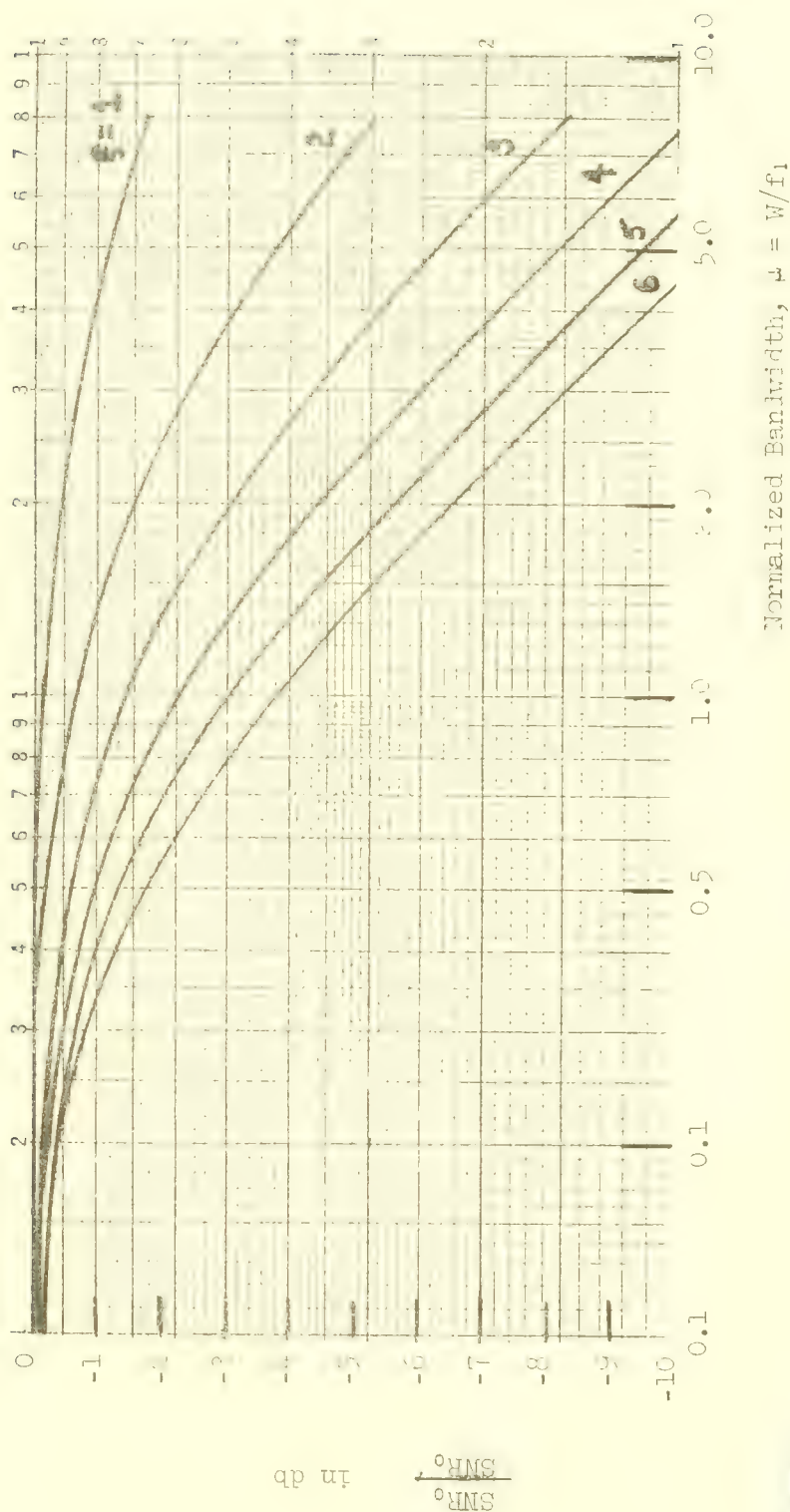Bandwidth Eckart System (SNR$_0$'(∞))

Figure 7 Comparison of Bandlimited Prewhitening System (SNRo) to Bandlimited Eckart System (SNRo')

VII.   THE MISMATCHED ECKART FILTER

   The relative performance of the pre-whitening system shown in
Figure 4 can be viewed as a plot of "mismatch" loss; i.e., if the system
bandwidth is matched to the signal spectrum ($\mu = \mu_0$ for each ξ), the loss
is 0db. For the simple spectral characteristics assumed in this report,
it is possible to make a similar plot demonstrating the effect of mismatch-
ing on the performance of an Eckart filter. The matching parameter for the
Eckart filter is, of course, not bandwidth, but the relative signal spectral
slope, $\bar{\xi}$. Consequently, we are interested in determining the performance
of an Eckart filter designed for a signal slope of ξ_0 in an environment
where the actual received signal has a slope of ξ, not necessarily equal
to ξ_0. The implications of mismatching for more realistic spectra are
discussed in the conclusions.

   The input signal and noise spectra are

$$\hat{S}_i(f) = \begin{cases} A f^{\alpha} & , \ f > f_1 \\ 0 & , \ f < f_1 \end{cases} , \ \alpha < 0 \tag{39a}$$

$$\hat{N}_i(f) = \begin{cases} B f^{\beta} & , \ f > f_1 \\ 0 & , \ f < f_1 \end{cases} , \ \beta < 0 \tag{39b}$$

The shaping filter has a transfer function

$$|\Gamma(f)|^2 = \frac{\hat{S}_i'(f)}{[\hat{N}_i(f)]^2} \tag{40}$$

where

$$\hat{S}_i'(f) = \begin{cases} A_0 f^{\alpha_0} & f > f_1 \\ 0 & , f < f_1 \end{cases} \tag{41}$$

Then the spectra into the square-law device are, by equation 13,

$$\hat{S}(f) = \frac{A A_0}{B^2} f^{-(\xi + \xi_0)} , \ f > f_1 \tag{42a}$$

$$\hat{N}(f) = \frac{A_0}{B} f^{-\xi_0} , \ f > f_1 \tag{42b}$$

where $\bar{\xi} = \beta - \alpha$, $\xi_0 = \beta - \alpha_0$. Applying equation 10 to these spectra, the
system output SNR is

$$SNR_0'(\xi_0) = T \left(\frac{A}{B}\right)^2 \frac{2\xi_0 - 1}{(\xi + \xi_0 - 1)^2} f_1^{1-2\xi} \tag{43}$$

$$\xi > \tfrac{1}{2}, \ \xi_0 > \tfrac{1}{2}$$

Referring to the matched Eckart filter result (equation 29), one obtains

$$\frac{SNR_o'(\xi_c)}{SNR_o'(\infty)} = \frac{(2\xi_o - 1)(2\xi - 1)}{(\xi + \xi_o - 1)^2} \quad , \; \xi > \tfrac{1}{2}, \; \xi_o > \tfrac{1}{2} \quad (44)$$

Note that for $\xi = \xi_0$, equation 44 reduces to unity, as it should.

Equation 44 is plotted in Figure 8. Cross reference to Figures 4 and 5 show how the performance of a mismatched Eckart system compares to a mismatched pre-whitening system. As an illustration, this has been done for two design values of ξ in Table I, and these indicate the Eckart system generally suffers less mismatching loss than the pre-whitening system under either base line comparison.
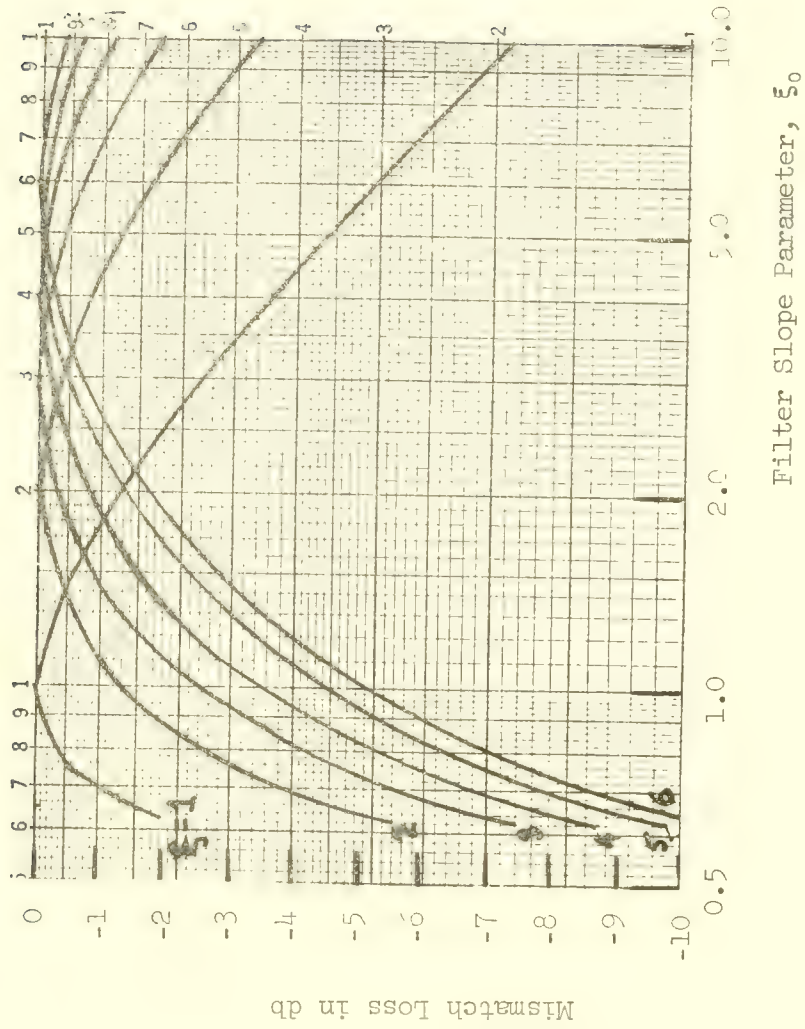
Figure 8   Mismatch Loss for Eckart Filter

TABLE I Comparison of Mismatching Losses

(a) Systems designed to optimize performance at $\xi = 1$

Actual value of ξ	Loss of pre-whitening system w.r.t optimum pre-whitening system (db)	Loss of pre-whitening system w.r.t. matched Eckart system (db)	Loss of mismatched Eckart system w.r.t. matched Eckart system (db)
1	0.0	-1.9	0.0
2	-2.0	-3.4	-1.3
3	-4.4	-5.5	-2.0
4	-6.4	-7.5	-3.6
5	-7.5	-8.7	-4.4
6	-8.8	-9.7	-5.2

(b) Systems designed to optimize performance at $\xi = 3$

Actual value of ξ	Loss of pre-whitening system w.r.t. optimum pre-whitening system (db)	Loss of pre-whitening system w.r.t. matched Eckart system (db)	Loss of mismatched Eckart system w.r.t. matched Eckart system (db)
1	-2.5	-4.4	-2.5
2	-0.4	-1.6	-0.4
3	0.0	-1.2	0.0
4	-0.2	-1.3	-0.2
5	-0.6	-1.6	-0.5
6	-1.1	-2.1	-0.7

VIII. CONCLUDING DISCUSSION

The specific results of this report are contained in Figures 4 through 8. Although some explanation and interpretation of these graphs was included in each of the individual sections where they initially appeared, this section will expand and clarify the results further. For convenience in reference, the five figures are reproduced at the end of this section.

Figure 4 shows that the penalty for using an incorrect bandwidth is not severe, though an intelligent choice of bandwidth can reduce this loss by a significant amount. As an example, assume that the signal spectra are expected to have relative slopes varying between -3 and -18db/oct. If the system used a bandwidth corresponding to $\mu = 5$, the loss (w.r.t. optimum μ's) in output SNR would vary between 0 and 10db. If the system were redesigned so that $\mu = 0.75$ was used, the loss would be between 0 and 2 db---an 8 db improvement in worst case system performance achieved very simply by choosing the "correct" bandwidth. If the occurrence of the various signal slopes were not equi-probable, the bandwidth choice might be altered. For example, if slopes of -3 and -6db/oct occurred 90% of the time, then one might choose $\mu \sim 2$ since the "majority" signals would suffer less than $\frac{1}{2}$ db of loss. With a μ of 2, the worst case loss is about $5\frac{1}{2}$ db more than the worst case loss associated with $\mu = 0.75$. But here, the worst case loss is suffered by the least probable signals, while with $\mu = 0.75$, the worst case loss of 2 db would be suffered by one of the more probable signals. All in all, of course, the final choice is a matter of the designer's judgement---no "rules" can be given; however, Figure 4 does contain information which should be useful in making such a judgement.

Figure 5 shows the additional losses entailed by not fully utilizing spectral information about the signal. Notice that if the region of concern were again $1 \leq \xi \leq 6$, the minimum worst case loss for a conventional bandlimited pre-whitening system no longer occurs at $\mu = 0.75$, but at $\mu = 0.9$; the change in the criterion of comparison has affected the choice of "correct" bandwidth, although not by a significant amount.

Naturally, as the region of signal interest changes, so does the choice of bandwidth; if the possible signal slopes lie between $3 \leq \xi \leq 5$ (between -9 and -15 db/oct), the bandwidth resulting in the smallest worst case loss would be $\mu \sim 0.43$. For the range of signal slopes plotted, the loss over and beyond the bandwidth loss of Figure 4 amounts to some 1 to 2 db.

Figures 6 and 7 show how the two preceding systems compare to a bandlimited Eckart filter. Figure 6 indicates that there is little use to extend the bandwidth of an Eckart filter beyond about 2 μ_0, where the μ_0 are optimum bandwidths determined from Figure 4. Figure 7 indicates that a narrow bandwidth Eckart filter, with $\mu \leq \mu_0/2$, is of little value since its performance is nearly identical to the simple bandlimited pre-whitening system.

Figure 8 shows the losses involved in using a mismatched Eckart filter, i.e., using a filter designed for ξ_0 when the signal spectrum actually has a slope ξ. By comparison with Figures 4 and 5, it can be seen that the Eckart filter generally exhibits less mismatching loss than the pre-whitening bandlimited system. If the designer were constrained to a single shaping filter specification, i.e., he could not implement a matching (Eckart) filter for each and every signal spectra, then a mismatched Eckart filter with $\xi_0 \sim 2.2$ would minimize the worst case loss for $1 \leq \xi \leq 6$. The performance of such a mismatched Eckart system is compared to the minimum worst-case, bandlimited, pre-whitening system in Table II. When the performance of both systems is compared to a matched Eckart filter, the mismatched Eckart is superior by some 1 to 2 db.

Thus, the following conclusions can be drawn from the numerical results contained in this paper. For signal spectra which have a constant slope w.r.t. noise of -10 log 2^{ξ} db/oct above some lower limit of f_1 cps:

(1) Any Eckart filter implementation need not have a bandwidth greater than 2 μ_0, where the μ_0 are determined from Figure 4.

(2) An Eckart filter, for any particular value of ξ, improves performance by 1 to 2 db over an optimized bandlimited pre-whitening system; although not totally insignificant, the improvement is marginal.

(3) A mismatched Eckart filter exhibits less loss than a mis-

TABLE II   Comparison of Minimum Worst Case Design to Optimum Systems

ξ	Loss of minimum worst case pre-whitening system w.r.t. optimum pre-whitening system (db)	Loss of minimum worst case pre-whitening system w.r.t. matched Eckart system (db)	Loss of minimum worst case mismatched Eckert system w.r.t. matched Eckart system (db)
1	-1.88	-3.38	-1.49
2	-0.10	-1.26	-0.05
3	-0.10	-1.37	-0.18
4	-0.61	-2.00	-0.61
5	-1.26	-2.68	-1.14
6	-1.88	-3.38	-1.49

matched pre-whitening system. If a minimum worst case loss criterion is used to specify μ, the bandwidth of the pre-whitening system, and ξ_0, the "matching parameter" of the Eckart filter, the Eckart design is superior by 1 to 2 db. Again, the improvement appears marginal.

The preceding results and conclusions are strictly valid only for the assumed simple spectral characteristics; however, the overall trends exhibited lead one to formulate hypothesized generalizations. The implications for conventional system design can be stated succinctly in one sentence: an intelligent choice of operating band is sufficient. The remainder of this section presents an expanded discussion of this thought.

This report basically constitutes a study of the significance of signal spectral characteristics upon the performance of a conventional passive detection system. Generally, the vicissitudes of the sonar environment are such that a detailed knowledge of signal characteristics is not possible. Under these conditions, an Eckart filter implementation is not practically feasible. The results in this report suggest two possible sub-optimum systems: the first is a conventional pre-whitening system with a bandwidth chosen to minimize the worst case loss over the expected range of signal variations; the second is a mismatched (band-limited) Eckart filter with a transfer function again chosen to minimize worst case loss. Of these two, the first is probably more practical. This is because realistic estimates of signal spectra are not susceptible to a simple character-ization, as by a single parameter ξ. Propagation effects can markedly alter the behavior of $S_1(f)$, and it would be difficult to make a plot such as Figure 8; because of the complicated behavior of $S_1(f)$, the phrase "mismatched Eckart filter" is almost meaningless. It is likely that there would exist a compromise filter that provides better worst case performance than the pre-whitening system, but the likelihood of such a filter being found by trial and error methods is probably small.* Further, the results in this report imply that the available improvement in performance is rather small. Consequently, there appears to be considerable justification for being content with the design of a conventional system under the constraint that the shaping filter perform only the function of pre-whitening the

*See Appendix D(2).

the noise and bandlimiting the input.

Note that within this constraint there are still considerable gains to be achieved by making intelligent use of signal spectral information. The numerical results contained in this report (Figures 4 and 5) quite possibly are not applicable to the realistic sonar situation, but the technique of analysis is. Thus, one could apply equations 1, 2, and 10 to more realistic estimates of the various sonar parameters ($T(f)$, $H(R,f)$, $N_s(f)$, etc.), examining the influence of perturbations in these quantities upon system performance (the output SNR) with different operating bands. Such a study would be more profitable than that carried out here since, first, the spectra would be more representative of the real world, and second, the study would determine both f_1 and W, i.e., both the location and width of the operating band. Such a comprehensive study may appear formidable and lengthy, but the utilization of digital computer techniques should reduce the effort required to an acceptable level.

Thus we return once more: an intelligent choice of operating band is sufficient. Such a conclusion has undoubtedly been bantered about intuitively for some time, so that some would question the value of the material contained in this report; however, the quantitative results presented here for simple spectra should provide a firmer basis for such an intuitive judgement, and as such, provide firmer ground for the conventional system designer to stand on.

APPENDIX A: SNR_0 EQUATION FOR SINGLE-SIDED POWER SPECTRA

The derivation of equation (9) utilized double-sided power spectra; however, in obtaining numerical results, it is often convenient to work with single-sided spectra. The necessary modification to equation 9 is derived below.

The double-sided result is (Eqn. 9)

$$SNR_0 = \frac{T}{2} \frac{\left[\int_{-\infty}^{\infty} S(f) df \right]^2}{\int_{-\infty}^{\infty} N^2(f) df}$$

Letting a super $\wedge$ denote single-sided spectra, we have the following conversions:

$$\int_{-\infty}^{\infty} S(f) df = \int_{0}^{\infty} \hat{S}(f) df$$

$$\int_{-\infty}^{\infty} N(f) df = \int_{0}^{\infty} \hat{N}(f) df$$

$$\therefore \quad \hat{N}(f) = \begin{cases} 2 N(f) & , f > 0 \\ 0 & , f < 0 \end{cases}$$

Thus

$$\int_{0}^{\infty} [\hat{N}(f)]^2 df = 4 \int_{0}^{\infty} N^2(f) df = 2 \int_{-\infty}^{\infty} N^2(f) df$$

since $N(f)$ is an even function. Thus

$$\int_{-\infty}^{\infty} N^2(f) df = \frac{1}{2} \int_{0}^{\infty} [\hat{N}(f)]^2 df$$

and equation 9 can be rewritten for single-sided spectra as

$$SNR_0 = T \frac{\left[\int_{0}^{\infty} \hat{S}(f) df \right]^2}{\int_{0}^{\infty} [\hat{N}(f)]^2 df} \quad .$$

APPENDIX B: GENERALIZATION OF EQUATION (11)

Because of its simplicity, equation (11) is particularly
appealing. It clearly reveals the small signal suppression effect --
the dependence of the output SNR upon the square of the input SNR --
which is characteristic of all non-linear detectors at low input SNR's
(4, pp 279-286; 14, pp 267 & 307). It also indicates the importance of
the "signal-processing" parameters, T and W. The general result in
equation (9) can be put in the form of the white noise result in equation
(11) by defining an appropriate equivalent noise bandwidth, viz.

$$W_e = \frac{1}{2} \frac{\left[\int_{-\infty}^{\infty} N(f)\,df\right]^2}{\int_{-\infty}^{\infty} N^2(f)\,df} \tag{B1.a}$$

or

$$W_e = \frac{\left[\int_0^{\infty} N(f)\,df\right]^2}{\int_0^{\infty} N^2(f)\,df} = \frac{\left[\int_0^{\infty} \hat{N}(f)\,df\right]^2}{\int_0^{\infty} [\hat{N}(f)]^2\,df} \tag{B1.6}$$

Then equation 9 can be written as

$$SNR_o = T W_e \left[\frac{\int_{-\infty}^{\infty} S(f)\,df}{\int_{-\infty}^{\infty} N(f)\,df}\right]^2 = T W_e \left(\frac{S}{N}\right)^2 \tag{B2}$$

where S and N denote total input signal and noise powers - i.e., the
result of integrating the input power spectra S(f) and N(f).

There is precedence for introducing a definition such as
equation B1: e.g., Blackman and Tukey (16, p 19 ff) utilize this defini-
tion of equivalent width in their analysis of power spectra measurement.
Although W_e may appear to be an obscure method of defining bandwidth,
rewriting and manipulating equation B1 will reveal a physical interpre-
tation of the result. Thus

$$W_e = \frac{1}{2} \frac{\int_{-\infty}^{\infty} N(\xi)\,d\xi \int_{-\infty}^{\infty} N(f)\,df}{\int_{-\infty}^{\infty} N^2(f)\,df} = \frac{1}{2} \frac{\int_{-\infty}^{\infty} N(\xi)\,d\xi \int_{-\infty}^{\infty} N(f\pm\xi)\,df}{\int_{-\infty}^{\infty} N^2(f)\,df}$$

$$= \frac{1}{2} \frac{\int_{-\infty}^{\infty} \left[\int_{-\infty}^{\infty} N(f\pm\xi) N(\xi)\,d\xi\right]\,df}{\int_{-\infty}^{\infty} N^2(f)\,df}$$

or

$$W_e = \frac{1}{2} \frac{\int_{-\infty}^{\infty} [N(f) \otimes N(f)]\,df}{N(f) \otimes N(f)\big|_{f=0}}$$

Aside from the factor of $\frac{1}{2}$, W_e is the ratio of the area under the autocorrelation (autoconvolution) function of $N(f)$ to the value of the autocorrelation function evaluated at zero shift. Thus, W_e can be referred to as the equivalent autocorrelation width of $N(f)$; Bracewell (17, pp 152-155) presents a discussion as to why such a "width measure" can be advantageous under certain conditions, although his measure does not include the factor of $\frac{1}{2}$.

The concept of an equivalent autocorrelation width has been freely used in the time domain, i.e. for $F_x(\tau)$ (9, pp 246-248); there, the width is referred to as the correlation time, τ_e, for the process $x(t)$. The actual correlation function, $R_x(\tau)$, is replaced by a rectangular function of width τ_e and height $R_x(0)$. It can readily be seen that an analagous interpretation is valid for the frequency domain; W_e is one of the many possible --- though perhaps least used -- definitions of band-width.

The factor of $\frac{1}{2}$ may be bothersome, but it is basically just fall-out from the mathematics. It could, of course, be arbitrarily left out of the definition in eqn. (B1), and a factor of $\frac{1}{2}$ would then appear in eqn (B2). It is more convenient and intuitively satisfying to leave it in the definition of bandwidth. As defined in (B1), both ideal low pass and ideal band pass (reactangular functions) white noise of bandwidth W have $W_e = W$. Similarly, a triangular spectrum of total extent W has $W_e = 0.75\,W$. Other examples are provided in both Blackman and Tukey (16) and Bracewell (17).

APPENDIX C: DEMONSTRATION THAT $\xi = \frac{1}{2}$ IS THE CRITICAL VALUE OF ξ

A convenient way to determine the critical value of ξ is to study both sides of the extrema equation, equation 18. Define the following two functions:

$$f_\xi(\mu) = (2\xi - 1)\mu + 1$$

$$g_\xi(\mu) = (1+\mu)^\xi$$

where $\xi \neq 1$, $\mu \geq 0$. Then equation 18 can be restated as

$$f_\xi(\mu) = g_\xi(\mu)$$

and the extrema of $\gamma_\xi(\mu)$ are determined by the intersections of $f_\xi(\mu)$ with $g_\xi(\mu)$. It is readily verified by substitution that $\mu = 0$ is always a solution, but since this corresponds to zero bandwidth, it is not the maximum of $\gamma_\xi(\mu)$ we are seeking. $f_\xi(\mu)$ is a straight line, and if $\xi < \frac{1}{2}$, it has a negative slope with an intercept of 1 at $\mu = 0$. Temporarily restricting ξ to be > 0, $g_\xi(\mu)$ is a "square root" type of curve, always greater than one, with a value of unity at $\mu = 0$. Consequently, $f_\xi(\mu)$ does not intersect $g_\xi(\mu)$ at any value of $\mu > 0$. Thus infinite bandwidth solutions are obtained for all values of $\xi < .\frac{1}{2}$. (The negative ξ, infinite bandwidth solutions are a trivial extension of the above arguments.)

For $\xi = \frac{1}{2}$, $f_{\frac{1}{2}}(\mu)$ is a line of zero slope, at a height of 1, and $g_{\frac{1}{2}}(\mu)$ is $\sqrt{1+\mu}$. Thus, in this case, there is no intersection (ignoring the trivial $\mu = 0$ conincidence), and the infinite bandwidth solution again results.

With $\xi > \frac{1}{2}$, $f_\xi(\mu)$ is now a straight line with positive slope. With $\frac{1}{2} < \xi < 1$, $g_\xi(\mu)$ is still a square root type of curve. With $\xi > \frac{1}{2}$, $f_\xi(\mu)$ is approaching ∞ as $\mu \longrightarrow \infty$ linearly with μ; with $\frac{1}{2} < \xi < 1$, $g_\xi(\mu)$ is approaching ∞ as $\mu \longrightarrow \infty$ as μ^ξ, which is less than linear with μ. Consequently, $f_\xi(\mu)$ will intercept $g_\xi(\mu)$ before μ becomes infinite.
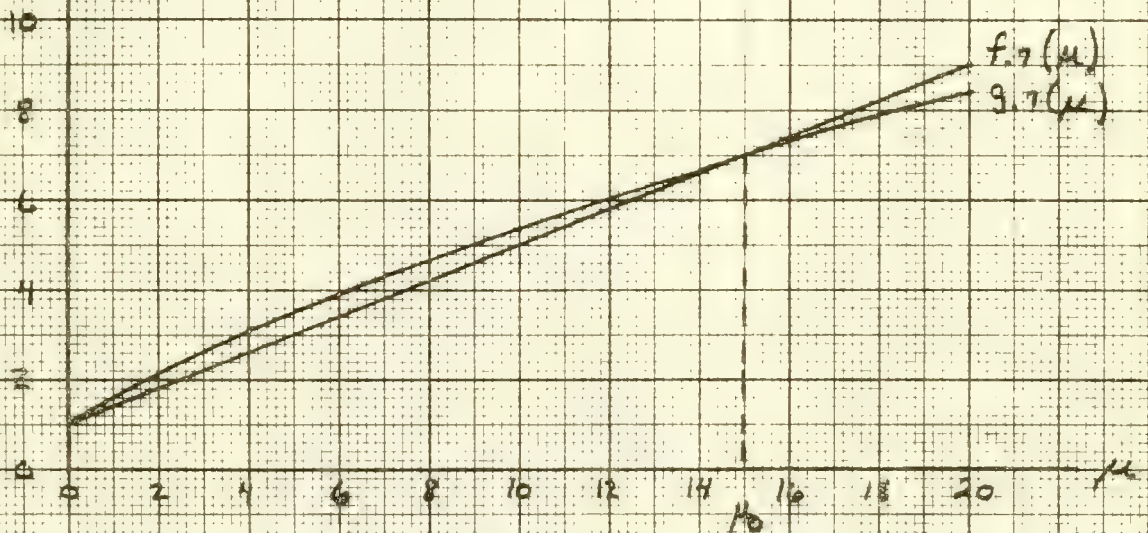
These concepts are indicated graphically in the accompanying figures for $\xi = 0$, 0.5, 0.7. Trial and error shows that the solution for $\xi = 0.6$ occurs at a value of μ approximately equal to 45.
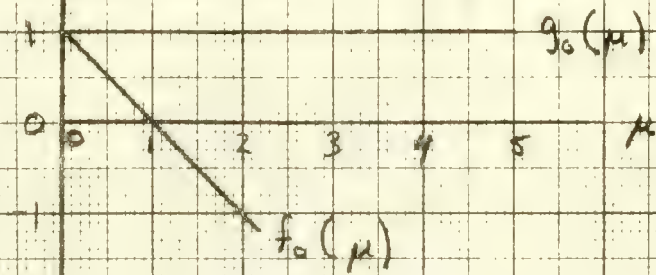
(a) $\xi = 0$
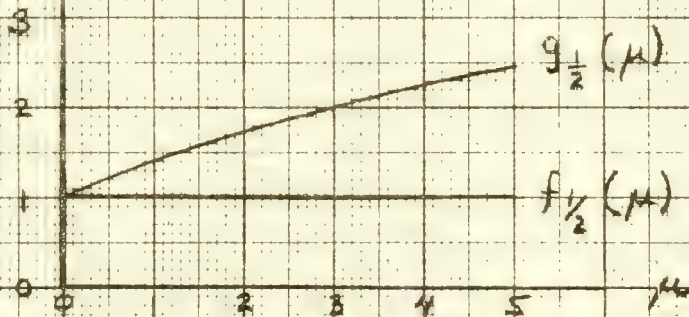
(b) $\xi = \frac{1}{2}$

(c) $\xi = 0.7$

Figure: Appendix C

APPENDIX D: ODDS AND ENDS

This appendix expands and clarifies some of the statements made in the text. The statements are of a general nature and rather lengthy, and, as such were deemed inappropriate for inclusion in the main body. The numbers of the headings refer to the footnote references in the text.

(1) That Bayes and maximum SNR lead to the same optimum configuration.

This statement is based upon a number of approximations and assumptions which may, at first glance, appear unjustifiable; however, experience indicates that the statement, though perhaps not rigorous, is reasonably correct. The logic leading to it is based on the following:

a. If the noise into the square-law device is bandlimited white and Gaussian, then the equi-spaced Nyquist samples of a segment T seconds long are independent and obey the chi-squared distribution at the output of the integrator (summer for sampled values). (1,pp 77-78)

b. For TW > 50, the chi-square distribution can be satisfactorily approximated as Guassian (18). This is simply an illustration of the Central Limit Theorem, but note that the chi-square distribution is sufficiently well behaved for the Gaussian approximation to be valid at rather small values of TW.

c. If the output of the processor is Gaussian, the statistical description of its performance is readily characterized by a single parameter d, defined as $d = \dfrac{\bar{z}(S+N) - \bar{z}(N)}{\sigma_z}$ where it is assumed that $\sigma_z = \sigma = \sigma_z(N) = \sigma_z(S + N)$. It will now be shown that maximization of this parameter results in a Bayes optimization for Guassian output statistics.

The system false alarm and detection probabilities are

$$P_{FA} = \int_{z_T}^{\infty} p_o(z)\, dz$$

$$P_D = \int_{z_T}^{\infty} p_1(z)\, dz$$

where Z_T = threshold value of Z, and

$$p_0(z) = (\sqrt{2\pi}\,\sigma)^{-1} \exp\left\{-\frac{(z-z_0)^2}{2\sigma^2}\right\}$$

$$p_1(z) = (\sqrt{2\pi}\,\sigma)^{-1} \exp\left\{-\frac{(z-z_1)^2}{2\sigma^2}\right\}$$

The subscripts 0 and 1 refer to N and S + N, respectively, and Z_0 and Z_1 are the output means.

Define two new variables,

$$\xi_0 = \frac{z - z_0}{\sigma} \qquad\qquad \xi_1 = \frac{z - z_1}{\sigma}$$

Then

$$P_{FA} = (\sqrt{2\pi})^{-1} \int_{\xi_{0T}}^{\infty} e^{-\xi_0^2}\, d\xi_0$$

where

$$P_D = (\sqrt{2\pi})^{-1} \int_{\xi_{1T}}^{\infty} e^{-\xi_1^2}\, d\xi_1$$

$$\xi_{0T} = \frac{z_T - z_0}{\sigma} \qquad\qquad \xi_{1T} = \frac{z_T - z_1}{\sigma}$$

If the P_{FA} is specified as some constant value, then ξ_{0T} is likewise a constant, say C_0, so that

$$\xi_{0T} = C_0 = \frac{z_T - z_0}{\sigma}$$

Thus

$$Z_T = \sigma C_0 + z_0$$

and

$$P_D = (\sqrt{2\pi})^{-1} \int_{C_0 - \frac{(z_1 - z_0)}{\sigma}}^{\infty} e^{-\xi^2}\, d\xi$$

Under the non-restrictive assumption $Z_1 > Z_0$, i.e., the mean with S + N is greater than the mean with N, the probability of detection is maximized by maximizing the quantity

$$d = \frac{z_1 - z_0}{\sigma}$$

Maximizing P_D while P_{FA} is held constant is just the Neyman-Pearson criterion, which differs from the Bayes criterion only in the specification of the threshold of the likelihood ratio (1, pp 69-70). Consequently maximizing d, or equivalently, maximizing the output SNR, satisfies the Bayes criterion for Gaussian output statistics.

It is important to realize the context in which these statements are made. In order for the maximization of d to have meaning, a process-or structure must be specified; one cannot derive the structure from d, but can optimally choose parameters of the structure by maximizing d. For an unstructured problem in which one is attempting to determine the best manner to process raw data, resort must be made to likelihood ratio techniques (the Bayes criterion).

The preceding logic is then applied to the problem at hand by stating that although the output statistics may not be chi-square (the noise is not necessarily white), they probably possess the characteristic of the chi-square distribution that the output distribution approaches Gaussian for reasonable values of TW_e, say TW_e greater than 100 or so. Then, by assuming small input SNR's, so that $\sigma_z(S + N) \sim \sigma_z(N)$, one can state that maximizing d (or the output SNR) satisfies the Bayes criterion.

(2) That the best one can do where highly limited a priori signal information is available is to whiten the noise and bandlimit the input.

The work contained in the body of this report can be considered an an attempt to justify this statement. The discussion in Section 8 is directly applicable; as stated there, it is likely that there exists a compromise filter, one which does not necessarily whiten the noise and would have a spectral shape somehow dependent upon the expected range of signal variations -- analogous to the mismatched Echart filter for simple spectra treated in Section 7 -- but that such a filter would be difficult to find since it would have to be determined by trial and error techniques. The quantitative results in this report indicate that the gains achievable from such a filter are minimal, and that a bandlimited pre-whitening system should prove satisfactory.

In the practical situation, of course, one is never going to build filters that actually whiten the noise or sharply bandlimit the input. The spectral fluctuations of the input noise and considerations of physical realizability theory make these idealistic concepts; the power of the dollar over system realization makes their approximations doubtful. The results in this report tend to substantiate the hope that these limitations do not severely compromise performance.

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